Linear, Exponential, and Logarithmic Functions

Slope y-intercept
Class Work

Identify the slope (m) and y-intercept (b) for each equation:

1. \( y = 3x - 4 \) 
2. \( y = -2x \) 
3. \( y = 7 \) 
4. \( x = -5 \) 
5. \( y = 0 \)

\[ m = \underline{\hspace{2cm}} \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad m = \underline{\hspace{2cm}} \]
\[ b = \underline{\hspace{2cm}} \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad b = \underline{\hspace{2cm}} \]

6. \( y - 3 = 4(x + 6) \) 
7. \( y + 2 = -0.5(x + 7) \) 
8. \( 2x + 3y = 9 \) 
9. \( 4x - 7y = 11 \)

\[ m = \underline{\hspace{2cm}} \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad m = \underline{\hspace{2cm}} \]
\[ b = \underline{\hspace{2cm}} \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad \quad b = \underline{\hspace{2cm}} \]

Write the equation of the given line from the graph to the right.

10. A ______________________
11. B ______________________
12. C ______________________
13. D ______________________
14. E ______________________
15. F ______________________

16. Write an equation for the following situation: Cal drives past mile marker 27 at 11am and mile marker 145 at 1pm.
Slope y-intercept

Homework

Identify the slope (m) and y-intercept (b) for each equation:

17. \( y = -5x - 2 \)  
18. \( y = 3x \)  
19. \( y = -2 \)  
20. \( x = 10 \)  
21. \( x = 0 \)

\[ m = \underline{\phantom{0000}} \quad m = \underline{\phantom{0000}} \quad m = \underline{\phantom{0000}} \quad m = \underline{\phantom{0000}} \quad m = \underline{\phantom{0000}} \]

\[ b = \underline{\phantom{0000}} \quad b = \underline{\phantom{0000}} \quad b = \underline{\phantom{0000}} \quad b = \underline{\phantom{0000}} \quad b = \underline{\phantom{0000}} \]

22. \( y - 4 = 2(x - 8) \)  
23. \( y + 3 = -0.4(x + 6) \)  
24. \( 3x + 4y = 9 \)  
25. \( 2x - 6y = 15 \)

\[ m = \underline{\phantom{0000}} \quad m = \underline{\phantom{0000}} \quad m = \underline{\phantom{0000}} \quad m = \underline{\phantom{0000}} \]

\[ b = \underline{\phantom{0000}} \quad b = \underline{\phantom{0000}} \quad b = \underline{\phantom{0000}} \quad b = \underline{\phantom{0000}} \]

Write the equation of the given line from the graph to the right.

26. A __________________________

27. B __________________________

28. C __________________________

29. D __________________________

30. E __________________________

31. F __________________________

32. Write an equation for the following situation: Jessie drives past mile marker 45 at 11am and mile marker 225 at 2pm.

Spiral Review

Factor:  
33. \( 3x^2 - 11x - 4 \)  

Simplify:  
34. \( \frac{-12x^6y^9}{8x^3y^3} \)  

Multiply:  
35. \( (2x - 3)(4x^2 - 2x + 3) \)  

Work out:  
36. \( (9x + 1)^2 \)
Different Forms of Lines

Class Work

The following equations of lines are in standard form. Find the x and y intercepts for each equation.
37. $2x + 3y = 12$  
38. $4x + 5y = 10$  
39. $x - 3y = 10$  
40. $4x = 9$  
41. $y = 0$

Write the equation for the described line in point-slope form.
42. Slope of 6 through (5, 1)  
43. Slope of -2 through (-4, 3)
44. Slope of 1 through (8, 0)  
45. Slope of $\frac{1}{2}$ through (1, -6)

Convert the following equations to both slope-intercept form and standard form.
46. $y - 4 = 5(x + 3)$  
47. $y = -2(x - 1)$  
48. $y + 7 = \frac{1}{5}(x - 8)$

Different Forms of Lines

Homework

The following equations of lines are in standard form. Find the x and y intercepts for each equation.
49. $3x - 5y = 15$  
50. $7x + 2y = 14$  
51. $x - y = 9$  
52. $y = 7$  
53. $x = 0$

Write the equation for the described line in point-slope form.
54. Slope of -4 through (4, -2)  
55. Slope of 3 through (0, -9)
56. Slope of $\frac{1}{2}$ through (6, 0)  
57. Slope of 2 through (5, -2)

Convert the following equations to both slope-intercept form and standard form.
58. $y - 3 = 7(x - 2)$  
59. $y + 1 = -4(x - 7)$  
60. $y + 3 = \frac{1}{8}(x - 8)$

Spiral Review

Simplify:
61. $(2x - 3)^3$  
62. $12x^4 - 38x^3 + 20x^2$

Factor:
63. $\frac{-24a^2b^2c^8}{8a^2b^4c^8}$

Simplify:
64. $\frac{a}{b^2}$
**Horizontal and Vertical Lines**

**Class Work**
Write the equation for the described line
65. vertical through (1,3)  
66. horizontal through (1,3)
67. vertical through (-2, 4)  
68. horizontal through (-2, 4)

**Horizontal and Vertical Lines**

**Homework**
Write the equation for the described line
69. vertical through (4,7)  
70. horizontal through (8,-10)
71. vertical through (8, -10)  
72. horizontal through (4, 7)

**Parallel and Perpendicular Lines**

**Class Work**
Write the equation for the described line:
73. Parallel to y= 3x + 4 through (1,3)  
74. Perpendicular to y= 3x + 4 through (1,3)
75. Parallel to y= -1/2x + 6 through (5, -2)  
76. Perpendicular to y= -1/2x + 6 through (5, -2)
77. Parallel to y = 5 through (-1,-8)  
78. Perpendicular to y = 5 through (-1,-8)

**Parallel and Perpendicular Lines**

**Homework**
Write the equation for the described line
79. Parallel to y= -2x + 1 through (1,-6)  
80. Perpendicular to y= -2x + 1 through (1,-6)
81. Parallel to y= 1/3x - 5 through (-5, 0)  
82. Perpendicular to y= 1/3x - 5 through (-5, 0)
83. Parallel to x = 5 through(-3, 7)  
84. Perpendicular to x = 5 through (-3,7)

**Spiral Review**
Simplify:  
85. \( \frac{x}{x^2} \)  
86. \((4x - 1)^2\)  
87. \((5x - 1)(3x^2 + 4x - 6)\)  
88. \(\frac{-24x^2y^{-4}x}{-2x^3yz^4}\)

Alg II: Linear, Exp, Log Functions

~4~
**Writing Linear Equations**

**Class Work**

Write an equation based on the given information. Use any form.

89. A line through (7, 1) and (-3, 4)
90. A line through (8, 2) and (8, -2)

91. A line perpendicular to \( y - 7 = 0.5(x + 2) \) through (-1, -8)

92. A line parallel to \( 4x - 7y = 10 \) through (2, 2)

93. A function with constant increase passing through (1, 3) and (8, 9)

94. The cost of a 3.8 mile taxi ride cost $5.50 and the cost of a 4 mile ride costs $5.70

95. A valet parking services charges $45 for 2 hours and $55 for 3 hours

**Writing Linear Equations**

**Homework**

Write an equation based on the given information.

96. A line through (4, 5) and (-5, -6)
97. A line through (-8, 2) and (8, 2)

98. A line perpendicular to \( 4x - 7y = 10 \) through (-1, -8)

99. A line parallel to \( y - 7 = 0.5(x + 2) \) through (2, 2)

100. A function with constant decrease passing through (1, 3) and (8, -9)

101. The cost of a 3.8 mile taxi ride cost $8.25 and the cost of a 4 mile ride costs $8.75

102. A valet parking services charges $55 for 2 hours and $75 for 4 hours

**Spiral Review**

Simplify: Work out: Multiply: Simplify:

103. \( \frac{\frac{3}{4}}{\frac{1}{4}} \) 104. \( 7 - 4(35 ÷ 5 \cdot 2) \) 105. \( (4x + 5)^3 \) 106. \( \frac{-12x^2y^4z^3}{-18x^4y^5} \)

Alg II: Linear, Exp, Log Functions ~5~
Identifying Exponential Growth and Decay

Class Work
State whether the given function is exponential growth or decay. Then find its horizontal asymptote and y-intercept.

107.

108.

109. \( y = 3(4)^x \)  
110. \( y = 0.5(3)^x \)  
111. \( y = 0.5^x + 4 \)

112. \( y = 2(0.25)^x - 7 \)  
113. \( y = 100(0.3)^x + 50 \)  
114. \( y = 17(4)^{-x} \)

115. \( y = 12(0.75)^{-x} + 6 \)

Identifying Exponential Growth and Decay

Homework
State whether the given function exponential growth or decay. Then find its horizontal asymptote and y-intercept.

116.

117.

118. \( y = 2(0.8)^x \)  
119. \( y = 3(5)^{-x} \)  
120. \( y = 4(0.3)^x + 2 \)

121. \( y = 3(15)^x - 2 \)  
122. \( y = 60(0.2)^{-x} + 20 \)  
123. \( y = 15(3)^x \)

124. \( y = 10(0.35)^x + 4 \)

Spiral Review
Multiply:  
124. \( (2x + 5)^2 \)  
Factor:  
125. \( 81x^2 - 36 \)  
Factor:  
126. \( 4x^2 + 25 \)  
Multiply:  
127. \( 5x^6(-3x^4y - x^3y^2) \)

Alg II: Linear, Exp, Log Functions  ~6~
Graphing Exponential Functions
Class Work
Graph each equation. Make sure that the y-intercept and the horizontal asymptotes are clear. Please number the axes on your graphs.

128. $y = 4(3)^x$

129. $y = 0.4(2)^x$

130. $y = (0.3)^x + 2$

131. $y = 3(0.45)^x - 2$

132. $y = 30(0.2)^x + 15$

133. $y = 12(3)^{-x}$

134. $y = 6(0.4)^{-x} + 2$
Graphing Exponential Functions

Homework

Graph each equation. Make sure that the y-intercept and the horizontal asymptotes are clear. Please number the axes on your graphs.

135. \( y = 4(0.3)^x \)

136. \( y = 4(4)^{-x} \)

137. \( y = 3(0.4)^x + 5 \)

138. \( y = 3(5)^x - 8 \)

139. \( y = 12(0.5)^{-x} + 30 \)

140. \( y = 10(5)^x \)

142. \( y = 2(0.45)^x + 3 \)

Spiral Review

Multiply:

143. \((3x - 4)^2\)

Simplify:

144. \(-\frac{15m^6n^3}{-5m^4n^7}\)

Factor:

145. \(125x^3 - 1\)

146. \(x^3 + 27\)
**Introduction to Logarithms**

**Class Work**

Write each of the following exponentials in logarithmic form.

147. \(10^2 = 100\) \hspace{1cm} 148. \(2^4 = 16\) \hspace{1cm} 149. \(27 = 3^3\)

Write each of the following logarithms in exponential form.

150. \(\log_5 125 = 3\) \hspace{1cm} 151. \(\log_6 36 = 2\) \hspace{1cm} 152. \(\log_7 343 = 3\)

**Solve the following equations**

153. \(\log_4 64 = x\) \hspace{1cm} 154. \(\log_2 64 = x\) \hspace{1cm} 155. \(\log_3 y = 5\)

156. \(\log_6 y = 3\) \hspace{1cm} 157. \(\log_b 81 = 4\) \hspace{1cm} 158. \(\log_b 1 = 10\)

**Introduction to Logarithms**

**Homework**

Write each of the following exponentials in logarithmic form.

159. \(9^2 = 81\) \hspace{1cm} 160. \(2^5 = 32\) \hspace{1cm} 161. \(81 = 3^4\)

Write each of the following logarithms in exponential form.

162. \(\log_8 64 = 2\) \hspace{1cm} 163. \(\log_4 256 = 4\) \hspace{1cm} 164. \(\log_3 81 = 4\)

Solve the following equations

165. \(\log_4 1024 = x\) \hspace{1cm} 166. \(\log_2 128 = x\) \hspace{1cm} 167. \(\log_5 y = 4\)

168. \(\log_7 y = 4\) \hspace{1cm} 169. \(\log_b 1000 = 3\) \hspace{1cm} 170. \(\log_b 1024 = 10\)

**Spiral Review**

171. Graph by hand: \(y = -|x - 2| - 1\) \hspace{1cm} 172. Graph by hand: \(y = -\sqrt{x + 1} - 3\) \hspace{1cm} 173. Factor: \(4x^2 - 9\) \hspace{1cm} 174. Multiply: \((3x + 1)(x^3 + 2)\)
Properties of Logs

Class Work

Using Properties of Logs, fully expand each expression.

175. \( \log_4 xy \)  
176. \( \log_3 x y^2 z^4 \)  
177. \( \log_{x^2} \frac{w}{x^2} \)

178. \( \log_5 \frac{1}{c^2 d^4} \)  
179. \( \log_7 \frac{7m^2}{(uv)^4} \)

Using Properties of Logs, rewrite expression as a single log.

180. \( \log x + \log y - \log z \)  
181. \( 2 \log c - 4 \log d \)  
182. \( 1 - 3 \log_5 m \)

183. \( 2 \log f + 3 \log g - 4 \log h \)  
184. \( 5 \log k - 3(\log r + \log t) \)

Properties of Logs

Home Work

Using Properties of Logs, fully expand each expression.

185. \( \log_4 x^5 y^2 \)  
186. \( \log_3 3 x y^2 z^5 \)  
187. \( \log_4 \frac{4w}{x^2} \)

188. \( \log_5 \frac{1}{c^2 d^3} \)  
189. \( \log_8 \frac{8m^4}{(u^2 v)^3} \)

Using Properties of Logs, rewrite expression as a single log.

190. \( 2 \log x + 3 \log y + 4 \log z \)  
191. \( 3 \log c - 5 \log d \)  
192. \( 1 - 2 \log_4 m \)

193. \( 5 \log f - 2 \log g - 6 \log h \)  
194. \( 3 \log k - 5 \log r + 5 \log t \)

Spiral Review

195. Graph by hand: \( y = -\log(x - 4) + 2 \)  
196. Graph by hand: \( y = -(x - 3)^2 - 5 \)  
197. Simplify: \( \frac{-6y^{-3}}{-36x^{-5}y^{-4}} \)  
198. Multiply: \( (8m^4n^3)(-4m^3n) \)

Alg II: Linear, Exp, Log Functions ~10~
Solving Logarithmic Equations
Class Work

Solve the following equations:

199. \( \log_5(x + 2) = \log_5(3x - 8) \)  
200. \( \log_4(3x - 6) = \log_4(x + 10) \)

201. \( \log_3x + \log_34 = 5 \)  
202. \( \log_2x + \log_2(x + 3) = 2 \)

203. \( 2\log_3x - \log_34 = 3 \)  
204. \( 3\log_5x - 3\log_54 = 4 \)

205. \( \log_3x + \frac{1}{2}\log_34 = \frac{1}{4}\log_316 \)  
206. \( \log_3x + \log_3(x - 2) = \log_335 \)

207. \( 2\log_3x - \log_34 = \log_316 \)  
208. \( \log_3(x + 3) + \log_3(x - 2) = \log_366 \)

Solve for the variable.

209. \( 7^x = 18 \)  
210. \( 4^{b-2} = 8 \)  
211. \( 5^{2d-3} = 29 \)

212. \( 7^{n-2} = 3^n \)  
213. \( 4^{t+2} = 5^{t-2} \)

Find the approximate value for each

214. \( \log_36 \)  
215. \( \log_517 \)  
216. \( \log_637 \)  
217. \( \log_9212 \)
Solving Logarithmic Equations

Home Work

Solve the following equations

218. $\log_5 (x - 2) = \log_5 (2x - 8)$

219. $\log_4 (2x + 7) = \log_4 (4x - 9)$

220. $\log_3 4x + \log_3 2 = 3$

221. $\log_2 x + \log_2 (x - 3) = 2$

222. $3\log_6 x - \frac{1}{2} \log_6 9 = 2$

223. $2\log_5 x - 2\log_5 5 = 3$

224. $\log_3 x - \log_3 4 = \log_3 16$

225. $\log_3 x^2 + \log_3 x = \log_3 27$

226. $2\log_3 x - \log_3 9 = \log_3 25$

227. $\log_3 (2x + 3) + \log_3 (x - 2) = \log_3 72$

Solve for the variable.

228. $8^x = 21$

229. $9^{b-6} = 42$

230. $19^{3d-1} = 40$

231. $2^{5-n} = 7^n$

232. $18^{t+1} = 32^{t-1}$

Find the approximate value for each:

233. $\log_3 10$

234. $\log_5 20$

235. $\log_6 30$

236. $\log_9 40$

Spiral Review

237. Find: $f \circ g$

If $g(x) = x^2 + 1$

and $f(x) = (2x + 3)^2$

238. Factor:

$81m^2 - 25n^2$

239. Simplify

$(-3x^2y^3)^3$

240. Describe the transformation:

$y = -|2x| - 1$

Alg II: Linear, Exp, Log Functions
e and ln

Class Work
Solve the following equations

241. \( e^{\ln x} = 6 \)  
242. \( e^{\ln x} - 4 = 6 \)  
243. \( \ln e^{x+5} = 6x \)

244. \( 3 \ln e^{2x} - 8 = 4 \)  
245. \( e^{2x} = 7 \)  
246. \( 3e^{(x-1)} + 9 = 10 \)

247. \( \ln(x + 1) = 7 \)  
248. \( \ln(x) + 1 = 7 \)

e and ln

Homework
Solve the following equations

249. \( e^{\ln 2x} = 6 \)  
250. \( 5e^{\ln x} - 4 = 6 \)  
251. \( \ln e^{2x-5} = 6 + x \)

252. \( 4 \ln e^{3x} + 9 = 21 \)  
253. \( e^{3x+1} = 6 \)  
254. \( 4e^{(2x+1)} + 8 = 10 \)

255. \( \ln(x - 1) = 9 \)  
256. \( \ln(x) - 1 = 9 \)

Spiral Review

257. Find: \( f \circ g \)
If \( g(x) = x^2 \)
and \( f(x) = 3x^3 - 1 \)

258. Factor: \( 27x^3 - 8y^3 \)

259. Simplify: \( (8x^3y^2)(-4x^4y^2)^2 \)

260. Describe the transformation: \( y = -|x + 2| - 3 \)
Growth and Decay
Class Work
Solve the following problems

261. $250 is deposited in an account earning 5% that compounds quarterly, what is the balance in the account after 3 years?

262. A bacteria colony is growing at a continuous rate of 3% per day. If there were 5 grams to start, what is the mass of the colony in 10 days?

263. A bacteria colony is growing at a continuous rate of 4% per day. How long till the colony doubles in size?

264. If a car depreciates at an annual rate of 12% and you paid $30,000 for it, how much is it worth in 5 years?

265. An unknown isotope is measured to have 250 grams on day 1 and 175 grams on day 30. At what rate is the isotope decaying? At what point will there be 100 grams left?

266. An antique watch made in 1752 was worth $180 in 1950; in 2000 it was worth $2200. If the watch’s value is appreciating continuously, what would its value be in 2010?

267. A furniture store sells a $3000 living room and doesn’t require payment for 2 years. If interest is charged at a 5% daily rate and no money is paid early, how much money is repaid at the end?
Growth and Decay

Homework

268. Solve the following problems
$50 is deposited in an account that earns 4% compounds monthly, what is the balance in the account after 4 years?

269. A bacteria colony is growing at a continuous rate of 5% per day. If there were 7 grams to start, what is the mass of the colony in 20 days?

270. A bacteria colony is growing at a continuous rate of 6% per day. How long till the colony doubles in size?

271. If a car depreciates at an annual rate of 10% and you paid $20,000 for it, how much is it worth in 4 years?

272. An unknown isotope is measured to have 200 grams on day 1 and 150 grams on day 30. At what rate is the isotope decaying? At what point will there be 50 grams left?

273. An antique watch made in 1752 was worth $280 in 1940; in 2000 it was worth $3200. If the watch’s value is appreciating continuously, what would its value be in 2010?

274. A $9000 credit card bill isn’t paid one month, the credit company charges .5% continuously on unpaid amounts. How much is owed 30 days later? (assume no other charges are made)

Spiral Review

275. Find the equation:

276. Find the equation:

277. Simplify:

$$\frac{-36a^2b^{-4}}{9a^{-4}b^5}$$
**Logistic Growth**

**Class Work**

Scientists measure a wolf population growing at a rate of 3% annually. They calculate the carrying capacity of the region to be 100 members.

278. Write a logistic equation to model this situation.

279. Create a table that shows the pack population over the next 10 years if $P_1 = 30$.

280. Draw a graph of the equation.

281. How long till the pack population is 60?

**Logistic Growth**

**Homework**

A calculus class determines that a rumor spreads around the school at a rate of 15% per hour. The school population is 1600.

282. Write a logistic equation to model this situation.

283. Create a table that shows the number of people who know the rumor if the class that starts it has 20 members.

284. Draw a graph of the equation.

285. How long till the majority of the school has heard the rumor?

**Spiral Review**

286. Factor:

287. Simplify:

288. Simplify:

289. Work out:

\[
8x^3 - 27 \quad -\frac{4xy^3z^3}{-10x^3y^2z^4} \quad \frac{4}{xyz} \quad \frac{3}{2x} + \frac{4y}{5}
\]
Multiple Choice

1. Which equation has an x-intercept of (5,0) and a y-intercept of (0, -2.5)
   a. \( y + 2.5 = 5(x - 0) \)
   b. \( y - 2.5 = 5(x - 0) \)
   c. \( y = \frac{1}{2}(x - 5) \)
   d. \( y = \frac{1}{2}(x + 5) \)

2. The equation of a line perpendicular to \( 2x + 3y = 7 \) and containing (5, 6) is
   a. \( 3x - 2y = 3 \)
   b. \( y - 6 = -\frac{2}{3}(x - 5) \)
   c. \( 3x - 2y = 4 \)
   d. \( y = \frac{2}{3}(x - 6) \)

3. Find the slope of a line parallel to the line \( 5x + 6y = 20 \)
   a. \( \frac{5}{6} \)
   b. \( -\frac{5}{6} \)
   c. \( \frac{6}{5} \)
   d. \( -\frac{6}{5} \)

4. A line with no slope and containing (3, 8) has equation
   a. \( y = 3 \)
   b. \( y = 8 \)
   c. \( x = 3 \)
   d. \( x = 8 \)

5. Which is the slope-intercept form of \( 7x - 4y = 8? \)
   a. \( y = \frac{7}{4}x + 2 \)
   b. \( y = -\frac{7}{4}x - 2 \)
   c. \( y = \frac{7}{4}x - 2 \)
   d. \( y = -\frac{7}{4}x + 2 \)

6. The standard form of \( y - 1 = -\frac{3}{7}(x + 2) \)
   a. \( y = -\frac{5}{4}x - 9 \)
   b. \( y = \frac{4}{5}x - \frac{9}{4} \)
   c. \( y = \frac{5}{4}x - \frac{9}{4} \)
   d. \( 4y = 5x - 9 \)

7. What is the equation of the line shown to the right?
   a. \( y = \frac{2}{3}x + 8 \)
   b. \( y = -\frac{2}{3}x + 8 \)
   c. \( (y - 8) = -\frac{2}{3}(x + 4) \)
   d. \( y + 2 = -\frac{2}{3}(x + 5) \)
8. The equation that models exponential decay passing through (0,5) and a horizontal asymptote of y = 4 is
   a. \( f(x) = 5e^x + 4 \)
   b. \( f(x) = -1e^x + 4 \)
   c. \( f(x) = 5e^{-x} + 4 \)
   d. \( f(x) = -1e^{-x} + 4 \)

9. A forest fire spreads continuously at a burning 10% more acres an hour. How long will it take for 1000 acres to be on fire after 200 acres are burning?
   a. 23.026 hours
   b. 16.094 hours
   c. 6.932 hours
   d. not enough information

10. \( \log_6 5 = \)
    a. .116
    b. .898
    c. 1.113
    d. 1.308

11. Evaluate \( \log_8 \frac{1}{2} \)
    a. \( \frac{1}{3} \)
    b. \( -\frac{1}{3} \)
    c. 3
    d. -3

12. Given \( 4^{x+1} = 10 \), find x
    a. 2.5
    b. .661
    c. .400
    d. 1.661

13. \( \log m = .345 \) and \( \log n = 1.223 \), find \( \log \frac{10m^2}{n^3} \)
    a. -1.979
    b. .651
    c. 6.507
    d. 8.473

14. Expand \( \log \left( \frac{ab^3}{10m^2} \right)^4 \)
    a. \( 4 \log a + 3 \log b - 8 \log m - 4 \)
    b. \( 4 \log a + 3 \log b - 8 \log m - 4 \)
    c. \( 4 \log a + 12 \log b - 8 \log m - 1 \)
    d. \( 4 \log a + 12 \log b - 8 \log m - 4 \)
15. Which of the following is equal to $5 \log a - 3 \log b - 4 \log c$?
   a. $\log \left( \frac{a^5}{b^3c^4} \right)$
   b. $\log \left( \frac{5a}{12bc} \right)$
   c. $-2 \log(abc)$
   d. $\log \frac{a^5c^4}{b^3}$

16. Solve: $3e^{(2x+1)} - 5 = 10$
   a. .305
   b. .609
   c. 1.305
   d. 2.61

17. Find the balance to the nearest dollar for $8000$ invested at a rate of 6% compounded for three years if the interest is compounded monthly.
   a. $65,178$
   b. $9573$
   c. $9528$
   d. $8121$

18. How much would you need to invest now at 7% compounded daily to have a balance of $1,000,000$ in 50 years?
   a. $30,208$
   b. $302,080$
   c. $33,898$
   d. $338,988$

19. A bacteria constantly grows at a rate of 20% per day. If initially there were 50, how long until there were 1000?
   a. 16.43 days
   b. 14.98 days
   c. .599 days
   d. 4.6 days

20. Which of the following would not influence the carrying capacity of a logistic growth model:
   a. the population of a town
   b. the food supply in an ecological preserve
   c. the rate of spread of the flu
   d. the area inside a Petri dish
**Short Constructed Response** – Write the correct answer for each question. No partial credit will be given.

1. The population of a country was 6 million in the year 2000 and has grown continually since then. The function \( P(t) = 6e^{0.016t} \), models the population, \( P \), in millions for \( t \) years since 2000.
   
   a. What is the estimated population at the end of the year 2013?

   
   b. In what year will the population reach 10 million?

2. Expand the following logarithm. Simplify where possible: \( \log \left( \frac{m^6}{10^p} \right)^4 \)

3. Make the following into one logarithm: \( 8\log_3 10 - 11\log_3 x - 3\log_3 y + \log_3 z \)

4. Solve: \( \log_6 (3x - 1) + \log_6 (x + 2) = \log_6 64 \)

5. Solve: \( 8^{3x-2} = 11^{2x} \)
**Extended Constructed Response** – Show all work. Partial credit may be given.

1. $50,000$ invested at an interest rate of $0.06$ percent compounded monthly can be represented by the function $A(t) = 50,000 \left(1 + \frac{0.06}{12}\right)^{12t}$.
   
   Use the equation above to answer the following questions.
   
   a) What will be the value of $A(t)$ after $4$ years?
   
   b) How long will it take for the initial amount to increase by $20,000$?

2. Entomologists introduce $20$ of one variety of insect to a region and determine that the population doubles every $6$ hours.
   
   a. Write an equation to model this situation. Assume that the population is continuously growing.

   b. What will the population be in $10$ days?

   c. How long will it take until the population reaches $100,000$?

3. A compostable bag breaks down such that only $10\%$ remains in $6$ months.
   
   a. If the decomposition is continual, at what rate is the bag decomposing?

   b. How much of the bag remained after $4$ months?

   c. When will there be less than $1\%$ of the bag remaining?
Linear, Exponential and Logarithmic Functions - Answer Key

1. \( m = 3, \ b = -4 \)
2. \( m = -2, \ b = 0 \)
3. \( m = 0, \ b = 7 \)
4. \( m \) is undefined, there is no \( y \)-intercept
5. \( m = 0, \ b = 0 \)
6. \( m = 4, \ b = 27 \)
7. \( m = -0.5, \ b = -5.5 \)
8. \( m = -2/3, \ b = 3 \)
9. \( m = 0, \ b = -2 \)
10. \( m = 0, \ b = 0 \)
11. \( m = 2, \ b = -12 \)
12. \( y = -\frac{2}{3}x + 5 \)
13. \( y = \frac{1}{4}x - 2 \)
14. \( y = 3x - 6 \)
15. \( x = 8 \)
16. \( y = 59x + 27 \)
   \( x = \) hours past 11am
   \( y = \) mile marker
17. \( m = -5, \ b = -2 \)
18. \( m = 3, \ b = 0 \)
19. \( m = 0, \ b = -2 \)
20. \( m \) is undefined, there is no \( y \)-intercept
21. \( m \) is undefined, every point on the line intercepts the \( y \)-axis
22. \( m = 2, \ b = -12 \)
23. \( m = -0.4, \ b = -5.4 \)
24. \( m = -3/4, \ b = 9/4 \)
25. \( m = 1/3, \ b = -2.5 \)
26. \( y = \frac{5}{4}x - 4 \)
27. \( y = -5x + 10 \)
28. \( x = -6 \)
29. \( y = -\frac{1}{3}x - 1 \)
30. \( y = \frac{1}{4}x - 2 \)
31. \( y = -6 \)
32. \( y = 60x + 45, \)
   \( x = \) hours past 11am
   \( y = \) mile marker
33. \((3x+1)(x-4)\)
34. \(-\frac{3xy^6}{2}\)
35. \(8x^3-16x^2+12x-9\)
36. \(81x^2+18x+1\)
37. \((6, 0)\) and \((0, 4)\)
38. \((2.5, 0)\) and \((0, 2)\)
39. \((10, 0)\) and \((0, -3.33)\)
40. \((2.25, 0)\) and none
41. N/A and \((0, 0)\)
42. \(y - 1 = 6(x - 5)\)
43. \(y - 3 = -2(x + 4)\)
44. \(y = x - 8\)
45. \(y + 6 = \frac{1}{2}(x - 1)\)
46. \(y = 5x + 19 \) and \(5x - y = -19\)
47. \(y = -2x + 2 \) and \(2x + y = 2\)
48. \(y = \frac{1}{5}x - 8.6 \) and \(x - 5y = 43\)
49. \((5, 0)\) and \((0, -3)\)
50. \((2, 0)\) and \((0, 7)\)
51. \((9, 0)\) and \((0, -9)\)
52. no \(x\)-intercept and \((0, 7)\)
53. \((0, 0)\) and N/A
54. \(y + 2 = -4(x - 4)\)
55. \(y + 9 = 3x\)
56. \(y = \frac{1}{4}(x - 6)\)
57. \(y + 2 = 2(x - 5)\)
58. \(y = 7x - 11 \) and \(7x - y = -11\)
59. \(y = -4x + 27 \) and \(4x + y = 27\)
60. \(y = \frac{1}{6}x - 4 \) and \(x - 6y = 26\)
61. \(8x^3 - 36x^2 + 54x - 27\)
62. \(2x^2(2x - 5)(3x - 2)\)
63. \(-\frac{3b^3}{a^4}\)
64. \(\frac{a^3d^3}{b^4c}\)
65. \(x = 1\)
66. \(y = 3\)
67. \(x = -2\)
68. \(y = 4\)
69. \(x = 4\)
70. \(y = -10\)
71. \( x = 8 \)
72. \( y = 7 \)
73. \( y = 3x \)
74. \( y = -\frac{1}{3}x + \frac{10}{3} \)
75. \( y = -\frac{1}{2}x + \frac{1}{2} \)
76. \( y = 2x - 12 \)
77. \( y = -8 \)
78. \( x = -1 \)
79. \( y = -2x - 4 \)
80. \( y = \frac{1}{2}x - \frac{13}{2} \)
81. \( y = \frac{1}{3}x + \frac{5}{3} \)
82. \( y = -3x - 15 \)
83. \( x = -3 \)
84. \( y = 7 \)
85. \( \frac{4}{3x^2} \)
86. \( 16x^2 - 8x + 1 \)
87. \( 15x^3 + 17x^2 - 34x + 6 \)
88. \( \frac{12}{x^4y^2z^3} \)
89. \( y = -\frac{3}{10}x + \frac{31}{10} \)
90. \( x = 8 \)
91. \( y = -2x - 10 \)
92. \( y = \frac{4}{7}x + \frac{6}{7} \)
93. \( y = \frac{6}{7}x + \frac{15}{7} \)
94. \( y = x + 1.7 \)
95. \( y = 10x + 25 \)
96. \( y = \frac{11}{9}x + \frac{1}{9} \)
97. \( y = 2 \)
98. \( y = -\frac{7}{4}x - 9.75 \)
99. \( y = .5x + 1 \)
100. \( y = -\frac{12}{7}x + \frac{33}{7} \)
101. \( y = \frac{5}{2}x - 1.25 \)
102. \( y = 10x + 35 \)
103. \( \frac{3x^3}{4} \)
104. \( -49 \)
105. \( 64x^3 + 240x^2 + 120x + 125 \)
106. \( \frac{2xz}{3y^5} \)
107. Decay
108. Growth
109. Growth
110. Growth
111. Decay
112. Decay
113. Decay
114. Decay
115. Growth
116. Growth
117. Decay
118. Decay
119. Decay
120. Decay
121. Growth
122. Growth
123. Growth
124. Decay
125. \( (9x + 6)(9x - 6) \)
126. Not factorable
127. \( 15x^{10}y + 5x^3y^2 \)
143. 9x^2 - 24x + 16
144. \frac{3n^4}{m^2}
145. (5x - 1)(25x^2 + 5x + 1)
146. (x + 3)(x^2 - 3x + 9)
147. \log 100 = 2
148. \log_2 16 = 4
149. \log_3 27 = 3
150. 5^3 = 125
151. 6^2 = 36
152. 7^3 = 343
153. x = 3
154. x = 6
155. y = 243
156. y = 216
157. b = 3
158. b = 1
159. \log_9 81 = 2
160. \log_2 32 = 5
161. \log_3 81 = 4
162. 8^2 = 64
163. 4^2 = 256
164. 3^4 = 81
165. x = 5
166. x = 7
167. y = 625
168. y = 2401
169. b = 10
170. b = 2

171. x^5
172. (2x + 3)(2x - 3)
173. \log x + 3 \log y + 4 \log z
174. \log x - 2 \log x
175. (1 + 2 \log_7 m) - 4(\log_7 u + \log_7 v)
176. \log_{5} 1 - (3 \log_5 c + 4 \log_5 d)
177. (1 + 2 \log_7 m) - 4(\log_7 u + \log_7 v)
178. \log_{5} \frac{c^2}{d^4}
179. \log_{5} \frac{5}{m^3}
180. \log_{5} \frac{f^2 g^3}{h^4}
181. \log_{4} x + 2 \log_4 y
182. \log_{4} \frac{4}{m^2}
183. \log_{4} \frac{4}{g^2 h^6}
184. \log_{4} \frac{k^5}{r^3}
185. 5 \log_4 x + 2 \log_4 y
186. 1 + \log_3 x + 2 \log_3 y + 5 \log_3 z
187. 1 + \log_4 w - 2 \log_4 x
188. -2 \log_5 c + 5 \log_5 d
189. 1 + 4 \log_8 m - 3(2 \log_8 u + \log_8 v)
190. \log_4 x^2 y^3 z^4
191. \log_2 \frac{c^2}{d^5}
192. \log_4 \frac{4}{m^2}
193. \log_4 \frac{4}{g^2 h^6}
194. \log_4 \frac{k^5}{r^3}
195. \frac{x^5 y}{6}
196. -32mn^4
197. x = 5
198. x = 8
199. x = 60.75
200. x = 1
201. x = 1
202. x = 29.39
204. \( x = 34.2 \)
205. \( x = 1 \)
206. \( x = 7 \)
207. \( x = 8 \)
208. \( x = 8 \)
209. \( x = 1.49 \)
210. \( b = 3.5 \)
211. \( d = 2.55 \)
212. \( n = 4.59 \)
213. \( t = 26.85 \)
214. \( 1.63 \)
215. \( 1.76 \)
216. \( 2.02 \)
217. \( 2.44 \)
218. \( x = 6 \)
219. \( x = 8 \)
220. \( x = 3.375 \)
221. \( x = 4 \)
222. \( x = 4.76 \)
223. \( x = 55.9 \)
224. \( x = 64 \)
225. \( x = 3 \)
226. \( x = 15 \)
227. \( x = 6.5 \)
228. \( x = 1.46 \)
229. \( b = 7.7 \)
230. \( d = .75 \)
231. \( n = 1.31 \)
232. \( t = 11.05 \)
233. \( 2.1 \)
234. \( 1.86 \)
235. \( 1.9 \)
236. \( 1.68 \)
237. \( f \circ g = 4x^4 + 20x^2 + 25 \)
238. \((9m+5n)(9m-5n)\)
239. \(-27x^8y^{2t} \)
240. H. shrink 0.5, flip x, ↓1
241. \( x = 6 \)
242. \( x = 10 \)
243. \( x = 1 \)
244. \( x = 2 \)
245. \( x = .97 \)
246. \( x = -0.1 \)
247. \( x = 1095.63 \)
248. \( x = 403.43 \)
249. \( x = 3 \)
250. \( x = 2 \)
251. \( x = 11 \)
252. \( x = 1 \)
253. \( x = .26 \)
254. \( x = -0.85 \)
255. \( x = 8104.08 \)
256. \( x = 22026.47 \)
257. \( f \circ g = 3x^6 - 1 \)
258. \((3x-2y)(9x^2+6xy+4y^2) \)
259. \(128x^{11}y^6 \)
260. \(-2, \text{ flip } x, \downarrow 3 \)
261. \$290.19 \)
262. 6.75 grams
263. 17.33 days
264. $15,831.96
265. 1%, Day 93
266. $3,315.40
267. $3,315.49
268. $61.04
269. 19.03 grams
270. 11.55 days
271. $13,122
272. 1%, Day 139
273. $4,604.50
274. $10,456.51
275. \( y = \sqrt{x+2} - 2 \)
276. \( y = (2-x)^3 + 1 \)
277. \( \frac{-4a^6}{b^9} \)
278. \( P(t) = \frac{100P_0e^{.03t}}{100+P_0(e^{.03t}-1)} \)
279. Create a table that shows the pack population over the next 10 years if $P_0 = 30$

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pop.</td>
<td>30</td>
<td>31</td>
<td>31</td>
<td>32</td>
<td>33</td>
<td>33</td>
<td>34</td>
<td>35</td>
<td>35</td>
<td>36</td>
<td>37</td>
</tr>
</tbody>
</table>

280.
281. 41.7 years
282. $P(t) = \frac{1600P_0e^{15t}}{1600+P_0(e^{15t}-1)}$

<table>
<thead>
<tr>
<th>Hour</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pop.</td>
<td>20</td>
<td>23</td>
<td>27</td>
<td>31</td>
<td>36</td>
<td>42</td>
<td>48</td>
<td>56</td>
<td>65</td>
<td>74</td>
<td>86</td>
</tr>
</tbody>
</table>

284.
285. 29.13 hours
286. $(2x-3)(4x^2+6x+9)$
287. $\frac{2y^5}{5x^2z^7}$
288. $\frac{4}{3yz}$
289. $\frac{15+8xy}{10x}$
Multiple Choice
1. c 11. b
2. a 12. d
3. b 13. a
4. b 14. d
5. c 15. a
6. c 16. a
7. c 17. b
8. c 18. a
9. b 19. d
10. b

Short Constructed Response
1. a. \[ P(13) = 6e^{0.016(13)} \]
   b. 7.387 million
   c. During the end of the year 2031 (31.93 years after 2000)

2. \[ 24 \log m - 28 \log n - 4 \]

3. \[ \log_3 \frac{10^8x^3}{x^{13}y^3} \]
4. \[ x = 3.93 \]
5. \[ x = 2.88 \]

Extended Constructed Response
1. a. \$63,524 (there could be some small variations due to rounding)
   b. 5.62 years

2. a. \[ A(t) = 20e^{2.77t} \] (Use the equation \[ 2P = Pe^{25r} \] to find \( r = 2.77 \))
   b. Approximately 21,428,000,000,000
   c. 3 days

3. a. 38% per month
   b. 22%
   c. After 12.12 months